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Computation of Various Ionospheric Constraints at GPS Operating Frequency Bands

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Abstract— The Global Positioning System (GPS) is a satellite based navigation system developed by the U.S. Department of Defense. The positional accuracy of Global Positioning System (GPS) is affected by several errors, the most predominant error being the ionospheric delay. This delay is proportional to the Total Electron Content (TEC). The accuracy provided by the standalone GPS is not sufficient for Precision Approach and landing requirements of civil aviation. In this paper, prominent ionospheric constraints affecting the performance of GPS are briefly described and typical values of these effects in the GPS operating frequency bands are computed. The estimation of ionospheric time delay using the Klobuchar model is presented.

Index Terms— Doppler Shift, Faraday Rotation, Global Positioning System, Group Delay, Klobuchar model, Phase Advance, Scintillations, Total Electron Content.

1 Introduction

THE Global Navigation Satellite System (GNSS) is a space-based radio positioning system that includes one or more satellite constellations capable of providing 24-hour three-dimensional position, velocity and time information to suitably equipped users anywhere on, or near, the surface of the earth. There are three prominent GNSS in the world. Out of these, the Global Positioning System (GPS) is the most well known and is operated by the U.S. Department of Defense. GPS was designed as a "dual-use" (military and civilian) system and achieved full operational capability (FOC) on July 17, 1995 with 24 Block II/IIA satellites. A second configuration called the Global Orbiting Navigation Satellite System (GLONASS) is maintained by the Russian Republic. GLONASS system consists of 24 operational satellites and has regained its FOC in December 2011 [1]. The third satellite navigation system currently under development is the Galileo system. The development of two regional navigation satellite systems by China and India is also under progress. The regional navigation system being developed in China is known as the Beidou system. The India's premier research organisation Indian Space Research Organization (ISRO) is developing the regional satellite navigation system named as Indian Regional Navigational Satellite System (IRNSS). The proposed Japanese Quasi-Zenith Satellite System (QZSS), is a three-satellite regional time transfer system and enhancement for GPS.

Over the past decade, the number of civilian applications of GNSS has increased significantly, as it provides reasonably good positioning accuracy in a cost effective manner. GPS is currently being used for a number of applications including aircraft and marine navigation, railway systems, fleet management, surveying and mapping, location based services and many other scientific applications. With the availability of multiple satellite constellations in the near future, the GNSS receiver would be capable of providing position information even in partially shadowed regions such as urban areas, forests, etc. However, the positional accuracy of GNSS is affected by several errors including ionospheric and tropospheric delay, satellite and receiver clock offsets, instrumental biases of the satellite and receiver, receiver measurement noise and multipath. The most predominant error among them is the ionospheric time delay error. In this paper, the prominent ionospheric constraints affecting the performance of GPS are briefly described and typical values of these effects in the GPS operating frequency bands are compared. The estimation of ionospheric time delay using the Klobuchar model is discussed with sample estimation results.

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2 Prominent Ionospheric Constraints

At frequencies below about 30 MHz the ionosphere acts almost like a mirror, reflecting the radio wave back towards the earth, thereby allowing long-distance communication. The lower limit of the usable radio-frequency spectrum is determined by ionospheric reflection and absorption which in general, increase with decreasing frequency [2]. At higher frequencies, such as those used in satellite radio navigation, radio waves penetrate through the ionosphere, but are modified by the medium due to presence of free electrons and earth's magnetic field [3]. The electron density is quantified by the number of electrons in a vertical column of cross sectional area of one square meter ($1m^2$), which is known as TEC. The TEC is a function of the amount of incident solar radiation. On the night side of the Earth, the free electrons have a tendency to recombine with the ions, thereby reducing the TEC. However, the TEC above a particular spot on the earth has a strong diurnal variation. There are also seasonal variations in the TEC and the variations that follow the Sun's 27-day rotational period and the roughly 11-year cycle of solar activity. The major effects of the ionosphere on GPS include refraction, group delay, phase advance, Faraday rotation, dispersion, Doppler shift and scintillations.

2.1 Refraction

The ionosphere changes the velocity of propagation of the GPS signals. It bends the GPS signal and changes its propagation speed, as the signal passes through the various layers in the ionosphere to reach the GPS receiver. The effect of signal bending causes a negligible range error, except at very low elevation angles less than 5°. It is the change in the propagation speed that causes a significant range error and should be accounted for [4]. The ionosphere acts as a dispersive medium for GPS signals. It means that the signal propagation velocity is a function of frequency.

2.2 Group Delay

As the GPS signals travel through the ionosphere, the signal modulating the carrier, i.e. the PRN codes and navigation message, propagate with a different speed that is less than the speed of light in free space. As a result, the code and navigation data stream is delayed by the ionosphere. The resulting group delay causes the measured range to be larger than the true range. The ionospheric group delay (in metres) is proportional to the total electron content (TEC) along the propagation path and is expressed as [5],

$$I_\rho = c\Delta\tau_g = \frac{40.3\ TEC}{f^2} \tag{1}$$

where TEC is the integrated electron density along the propagation path from the satellite to the receiver (electrons/m^2); f is the frequency in Hertz; $\Delta\tau_g$ is the group delay in seconds. The group delay affects the accuracy of satellite navigation systems.

2.3 Phase Advance

In the ionosphere, the carrier wave travels with a phase velocity that is larger than the speed of light in free space. The refractive index is slightly less than unity (~0.99998 for the GPS L1 frequency). Therefore, the measured range using the carrier phase will be shorter than the true range. The phase advance relative to that in the free space is given as [6],

$$\Delta\phi_p = \frac{40.3\ TEC}{cf} \tag{2}$$

or $$\Delta\phi_p = \frac{2\pi \times 40.3\ TEC}{cf} \text{ radians} \tag{3}$$

The group delay ($\Delta\tau_g$) is related to the phase advance ($\Delta\phi_p$) as,

$$\Delta\phi_p = -f\ \Delta\tau_g \tag{4}$$

i.e. for every cycle of carrier phase advance, there are (1/f) seconds of time delay. The minus sign in (4) indicates that the code group delay and carrier phase advance move in opposite directions. The importance of this effect is manifested when determining space object velocities by means of range rate measurements.

2.4 Faraday Rotation

Faraday rotation is brought about by the interaction of the electromagnetic waves with the free electrons in the presence of Earth's magnetic field. It alters the polarization of the GPS signal. In general, TEC data available throughout the world is derived from Faraday rotation measurements from various geostationery satellites. Due to Faraday rotation even though a linearly polarized signal is transmitted from satellite to the user, no signal will be received at the receiver's linearly polarized antenna, unless the receiving antenna polarization is realigned properly for maximum receiver signal.Due to this effect, a linearly polarized signal may become elliptically or circularly polarized. The Faraday rotation is directly proportional to TEC, and the component of the earth's magnetic field along the propagation path. It is inversely proportional to the square of the frequency.

For a path length of Z_p meters (in the ionosphere) the rotation angle Ω (in radians) is given by [7], [8],

$$\Omega = \int_0^z \frac{2.36 \times 10^4}{f^2} Z_p N B_o \cos\theta\, dz$$
$$\approx 1.885\ f^{-2}\ TEC \tag{5}$$

where θ is the angle between geomagnetic field and the direction of propagation. N is the electron density measured in electrons/m^3. B_o is the geomagnetic flux density in Teslas.

For satellite navigation applications, circular polarization is used to counter the problem of Faraday rotation. The GPS signals are transmitted with right-handed circular polarization (RHCP). Therefore, Faraday rotation has insignificant impact on the GPS signal.

2.5 Dispersion

Dispersion is due to the differential propagation delay of the signal frequencies, especially when significant bandwidth is involved. Dispersion is defined as the rate of change of time delay with frequency. The dispersion (in seconds/hertz) is expressed as [8],

$$\frac{d\Delta\tau_g}{df} = \frac{80.6\ TEC\ f^{-3}}{c} \tag{6}$$

Phase dispersion refers to the rate of change of phase with frequency. The phase dispersion (in radians/seconds) is given by [9],

$$\frac{d\Delta\phi_p}{df} = -80.6\pi\ f^{-2}\ TEC \tag{7}$$

The ionosphere causes a dispersion of the spread spectrum GPS signals as they pass through, which manifests itself as a difference in pulse arrival time across the 20MHz bandwidth. This effect is very small and is ignored.

2.6 Doppler Shift

The relative motion between the satellite and the receiver causes a shift in the frequency of the signal broadcast by the satellite, and is commonly known as Doppler shift. This Doppler shift is measured on the signal from each satellite in order to calculate the user velocity [10]. As seen earlier, the ionosphere causes a phase advance and because frequency is simply the time derivative of phase, an additional contribution to measured Doppler frequency shift results because of changing TEC. This additional frequency shift (in Hertz) can be computed as [6],

$$\Delta f_d = \frac{40.3}{cf} \times \frac{d}{dt}(TEC) \quad (8)$$

Parkinson and Spilker [11] reported that the upper limit to the rate of change of TEC for a stationary user is approximately 0.1×10^{16} (electrons/m^2) per second. At L1 frequency, this value yields an additional frequency shift of 0.085 Hz, which would be insignificant for most of the GPS applications.

2.7 Scintillations

Scintillations are variations of amplitude, phase, polarization and angle of arrival produced when radio waves pass through electron density irregularities in the ionosphere [12]. When the GPS satellite signal travels through these irregularities, it experiences both amplitude and phase scintillations. The scintillation noise comes under multiplicative noise. Mere increasing the transmitter power cannot improve the signal to noise ratio.

a) ***Amplitude Scintillation:*** The small-scale irregularities in the electron density diffract and refract the GPS signal, diverting radiated power away from the direction of the user; thus reducing the total signal power received by the user. This effect causes rapid random fluctuations in the signal intensity at the receiver and is known as amplitude scintillation. Amplitude fading could drop the signal-to-noise-ratio (SNR) below the receiver's threshold, causing a loss of code lock. This reduces the number of satellites available to perform a position solution. The timing of strong scintillation effects observed in the near equatorial regions are generally limited to the local late evening hours, approximately 1hour after local sunset to local midnight. Amplitude scintillation is measured by a parameter known as S_4 index, which is essentially a normalized standard deviation in the signal intensity over 60 seconds. The frequency dependence of moderate amplitude scintillation varies as $f^{-1.5}$ on an average over a range of frequencies between VHF and L-band. As the scintillation intensifies, the amplitude scintillation drops off more gradually with frequency, and the parameter S_4 tends toward unity. Strong amplitude scintillations are observed at frequencies atleast as high as 10 GHz in Japan and at 4 GHz in India [11].

b) ***Phase Scintillation:*** A GPS receiver uses the carrier phase information to maintain a lock on the GPS signal. Under normal ionospheric conditions, the carrier phase remains constant. After the GPS signal travels through the region of ionospheric irregularities, the phase of the carrier wave changes rapidly due to the presence of fluctuations in the electron density. This causes a Doppler shift in the frequency of the received carrier signal, which may exceed the bandwidth of the phase lock loop (PLL) in the receiver. As a result of this, the receiver could loose lock of the carrier phase. The rapid random fluctuation in the phase of the received carrier is known as phase scintillation [13]. Phase scintillation is quantified by a parameter $\sigma_{\Delta\varphi}$, which is a standard deviation of the detrended phase over an interval of up to 60 seconds. Normally, strong phase effects occur over mid latitudes and near-equatorial latitudes [6]. The phase scintillation varies as f^{-1}.

In the equatorial regions, scintillation effects are observed typically in the evening and nighttime. Severe scintillations of the order of 20 dB can cause loss of signal for one or more satellites, thus potentially degrading the navigation solution [14]. Thomas *et al* [15] showed that individual positional errors can increase from a few meters to over 30 m during GPS scintillation at low latitudes.

2.8 Computation of Ionospheric Constraints

Table 1 gives values of various ionospheric effects computed at the GPS operating frequency bands. It is assumed that the total electron content (TEC) is 5×10^{17} electrons/m^2. It is found that out of all the effects, the ionospheric time (group) delay is the most significant effect. This error can be of the order of 5m to 15m.

TABLE 1
VARIOUS IONOSPHERIC EFFECTS COMPUTED AT GPS OPERATING BANDS

S.No.	Ionospheric Effect	Error range at GPS operating frequency bands		
		L1 (1560.10 – 1590.80 MHz)	***L2 (1212.25 – 1242.95 MHz)***	***L5 (1161.10 – 1191.80 MHz)***
1.	Group delay (m)	7.96 – 8.27	13.04 – 13.71	14.18 – 14.94
2.	Phase advance (cycles)	42.22 – 43.05	54.03 – 55.40	56.35 – 57.84
3.	Faraday rotation (rad)	0.3724 – 0.3872	0.6101 – 0.6414	0.6636 – 0.6991
4.	Dispersion (sec/Hz)	$(3.33 – 3.53) \times 10^{-17}$	$(6.99 – 7.54) \times 10^{-17}$	$(7.93 – 8.58) \times 10^{-17}$
5.	Phase dispersion (rad/sec)	(-) 50.00 – (-)51.99	(-) 81.90 – (-) 86.10	(-) 89.08 – (-) 93.86
6.	Doppler shift (Hz)	0.0844 – 0.0861	0.1081 – 0.1108	0.1127 – 0.1157

3 Estimation of Ionospheric Time Delay using Klobuchar Model

The ionospheric time delay is one of the largest sources of error in space based navigation systems. The propagation effects of ionosphere are very complex. The errors contributed by the ionosphere for a single frequency user can be reduced, though not completely eliminated by the use of appropriate models. Klobuchar has developed an ionospheric time delay estimation algorithm for single frequency GPS users [16]. The model consists of a time delay algorithm that uses the positive portion of a cosine wave during daytime and a constant offset during night time to model the diurnal behaviour of vertical time delay as a function of the time of the day. The period of the best fit is not 24 hours but is the period required to minimize the rms distance between the actual monthly mean diurnal curve and the cosine fit. The equivalent period is larger than 24 hours.

The cosine function is given by [17],

$$T_g = DC + A * \cos 2\pi(t - T_p)/P \text{ seconds} \quad (9)$$

The expansion of cosine term results to,

$$T_g = DC + A(1 - 0.5x^2 + x^4/24) \quad (10)$$

where $|x| \le \pi/2$ seconds and x = 2π(t-T_2) / P radians; t = local time of earth sub-ionospheric point. The sub-ionospheric point is the signal ray intersection with the mean ionospheric height of 350 km. DC and phasing term (T_P) are held constant at 5ns and 14h (50400s) local time respectively. Amplitude (A) and period (P) can be modeled as third order polynomials.

The amplitude and period of the cosine function as a function of geomagnetic latitude are represented by third-order polynomials. The model can be expressed as,

$$\Delta T_{iono} = A_1 + A_2 \cos[(2\pi(\tau - A_3))/A_4] \quad (11)$$

where,

A_1 : Night-time value (5 x 10^{-9}) s

Amplitude, $A_2 = \sum_{n=0}^{3} \alpha_n \phi_m^n = (\alpha_1 + \alpha_2\phi_m + \alpha_3\phi_m^2 + \alpha_4\phi_m^3)$

A_3 : Phase at (1400 hrs LT) and Period,

$$A_4 = \sum_{n=0}^{3} \beta_n \phi_m^n = (\beta_1 + \beta_2\phi_m + \beta_3\phi_m^2 + \beta_4\phi_m^3)$$

ϕ_m : Geomagnetic latitude of the sub-ionospheric point

The Klobuchar coefficients (i.e.) α's and β's are eight polynomial coefficients uploaded by GPS master control station (MCS) to the GPS satellites. These coefficients were computed from an empirical model of worldwide ionospheric behavior derived by Bent. The set of coefficients for upload to the satellite is selected on the basis of the seasonal effects and solar flux evaluated at the time of the upload. The GPS satellites, in turn, transmit these coefficients down to the user in the RINEX Navigation message. The algorithm described, however, is a compromise of several factors such as the present state of knowledge of temporal, diurnal and geographic variations of TEC.

Steps in the estimation of ionospheric delay:

For a particular GPS satellite the Elevation and Azimuth angles be *E* and *Az* respectively, observed at the user location (ϕ_u, λ_u). The steps involved in computation of ionospheric delay are as follows. All angles are in the unit of half circles.

(i) Calculate Earth centered angle: $\psi = \frac{0.0137}{E + 0.11} - 0.022$ (semi-circles).

(ii) Calculate the sub-ionospheric latitude: $\Phi_l = \Phi_u + \psi \cos A$

(iii) Now, if $\Phi_l > +0.416$, then $\Phi_l = +0.416$ and if $\Phi_l < -0.416$, then $\Phi_l = -0.416$.

(iv) Compute the sub-ionospheric longitude: $\lambda_i = \lambda_u + \frac{\psi \sin A}{\cos \Phi_l}$.

(v) Geomagnetic latitude can be computed as: $\Phi_m = \Phi_l + 0.064 * \cos(\lambda_i - 1.617)$).

(vi) Find the local Time: $t = 4.32 \times 10^4 \lambda_i + \text{GPStime(sec)}$.

(vii) If, t >86400, use t = t-86400. If t < 0, t = t+86400.

(viii) Calculate the Slant Factor: $F = 1.0 + 16.0 \times (0.53 - E)^3$

Lastly, the computed ionospheric delay at L_1 frequency is given by,

$$T_{iono} = F * [5 * 10^{-9} + \sum_{n=0}^{3} \alpha_n \Phi_m^n * (1 - \{x/2\} + \{x^2/24\})]$$

where $x = \frac{2\pi(t - 50400)}{\sum_{n=0}^{3} \beta_n \Phi_m^n}$ (12)

The ionospheric delay at L_2 frequency can be estimated by multiplying the result for L_1 by 1.65.

Example estimation of ionospheric delay:

For a GPS satellite (PRN 13), observed on 10th Dec 2001 from Hyderabad (17.431°E, 78.45°N), the central angle ψ = 0.02899 semicircles (5.2°) , ϕ_I= 16.1025° and λ_I = 83.72°, for t = 84600 and F= 3.75, we get T_{iono} = 35.3 nsec (= 10.59 m).

The Klobuchar's ionospheric time-delay algorithm for single frequency GPS users (L_1) was aimed to minimize the user computational complexity and provides certainty to the extent of 50% of actual ionospheric error. It may be due to application of truncated sine/cosine representation of diurnal curve, reduction in number of trigonometric calculations as a result of putting approximations in the model. Comparing the algorithm against actual ionospheric TEC data, it has been shown that an overall reduction in rms error could be up to 60% in the northern hemisphere. The accuracy varies with the data interval over which the measurements are averaged (reflecting variation with the 11-yr solar-flux cycle as well as season of the year), location (reflecting variation with the geomagnetic latitude), local time of the day, satellite elevation angle, etc., giving a minimum average of 50% rms error reduction.

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The dual frequency GPS receivers can estimate the ionospheric delay with better accuracy as compared to single frequency receivers. These are more sophisticated and relatively expensive.

4 Conclusions

The standalone GNSS is affected by several errors, most predominant being the ionospheric delay. The various ionospheric effects at GPS frequencies are briefly described. Typical values of these effects in the GPS operating frequency bands are computed and presented in a tabular form. Out of all the ionospheric effects, the group delay is the most significant and is a function of the total electron content (TEC). This delay can cause a range error of about 10 to 15 m in the positioning solution. Klobuchar's single frequency algorithm is described with an example estimation. The dual frequency GPS receivers can be used to estimate the ionospheric delay with better accuracy for equatorial and low latitude regions such as India.

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